Optimal Evading Strategies and Task Allocation in Multi-Pursuer Single-Evader Problems

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Outline

- Motivation and problem statement
- Optimal evading strategies
- Active/Redundant pursuers
- Simulations

Motivation

- Airspace security
- Regulate the traffic and usage of UAVs

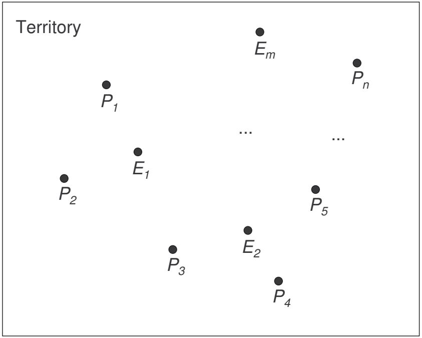
Figure 1: DroneHunter [a]

[a] https://fortemtech.com/

A scenario

Assume:

- n agents (pursuers) guarding a territory
- m adversaries (evaders, typically $m \leq n$)
- Pursuers want to capture the evaders
- Pursuers are faster than the evaders



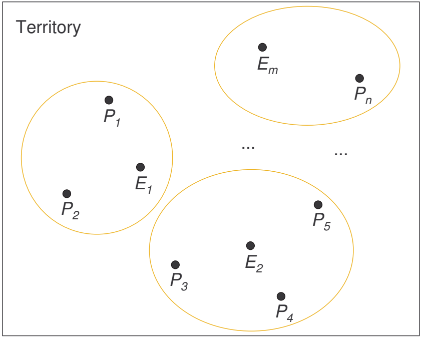
Some questions!

Relevant Questions:

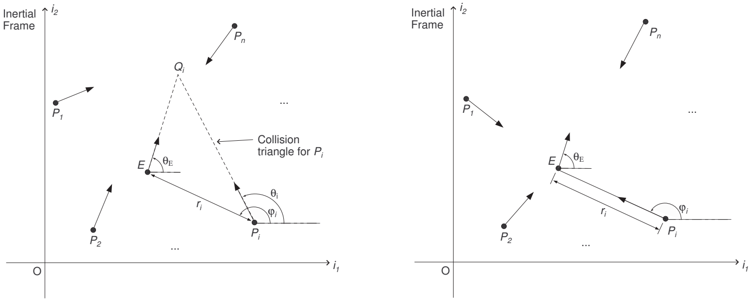
- What is the shortest time-to-capture, while evaders will try to postpone capture indefinitely?
- Which pursuer(s) should go after which evader(s)?
- A multi-pursuer multi-evader game!

Approach

- *Divide and Conquer*
- Solve m multi-pursuer single-evader games
- Pursuers follow simple navigation laws: *Pure Pursuit (PP)* or *Constant Bearing (CB)* staregies



Problem Set Up

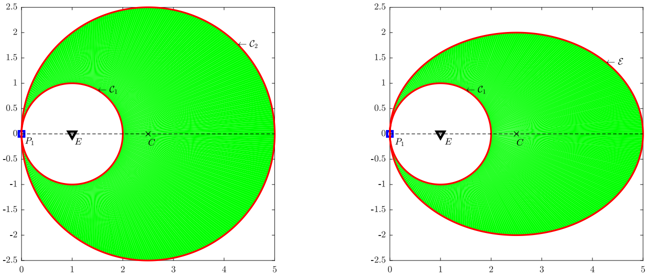


(a) CB (b) PP

Figure 2: Schematics of the proposed pursuit-evasion problems.

Identical pursuers, pursuers faster than evader.

Regions of Non-Degeneracy[1]



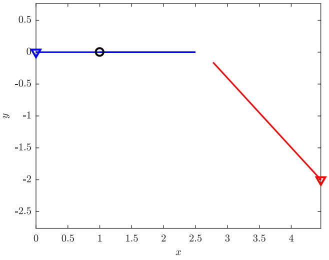
(a) CB

(b) PP

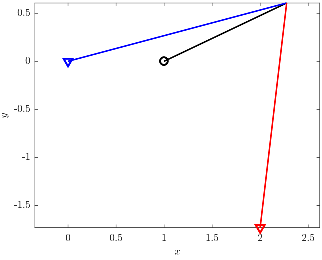
Figure 3: Regions of non-degeneracy

[1] Makkapati et al., *Pursuit-Evasion Problems Involving Two Pursuers and One Evader*, AIAA GNC Conference, Kissimmee, FL, 2018

Two Pursuers - CB (Previous work)



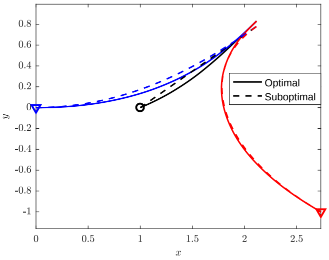
(a) A degenerate case



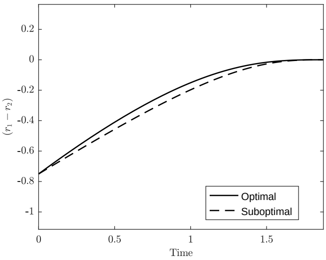
(b) A non-degenerate case

Figure 4: Trajectories of the players for optimal control inputs in Scenario 1: black - evader, blue - P_1, red - P_2.

Two Pursuers - PP (Previous work)



(a) Trajectories



(b) Time variation of difference in relative distances

Figure 5: Performance of the optimal and suboptimal strategies for a non-degenerate case in Scenario 2: black - evader, blue - P_1, red - P_2.

Optimal Evading Strategies

In both CB and PP cases:

Proposition

The time-optimal evading strategy is dependent only on the initial positions of those **pursuers that (simultaneously) capture the evader***.*

Let's call them the "influential" pursuers!

Some Issues

In both cases

- No analytical expression for the optimal strategy of the evader
- Hard to identify the influential pursuers - no theoretical backing!

What If?

- The pursuers don't know the evader's strategy

Capturing Pursuer Set

Definition

Given the initial positions of the players (at $t = 0$) in an MPSE problem and assuming that the pursuers follow either a CB or a PP strategy, for a given evading strategy, **the capturing pursuer set $\mathbb{P}$ is the set of pursuers that are in the capture zone of the evader at the time of capture (t_c)**.

Active/Redundant Pursuers

At time $0 \leq t < t_c$

Definition

If there exists an evading strategy for which pursuer P_i is in $\mathbb{P}$, then P_i is an **active pursuer**.

Definition

If there exists no evading strategy for which pursuer P_i is in $\mathbb{P}$, then P_i is a **redundant pursuer**

Apollonius Curves

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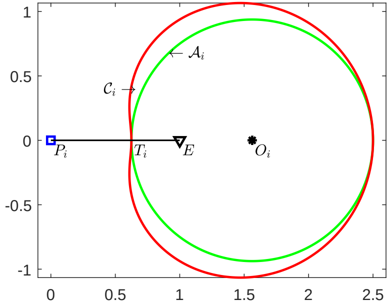
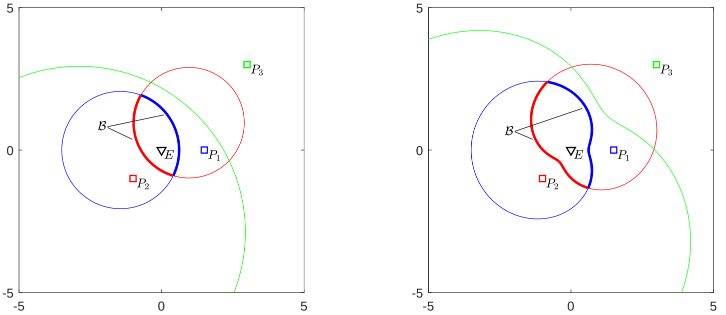


Figure 6: The locus of capture points for a non-maneuvering evader in the cases CB and PP. Simulation parameters: $u = 1$, $v = 0.6$.

Apollonius Boundary



(a) CB (b) PP

Figure 7: Apollonius boundaries for CB and PP cases (Simulation parameters: $u = 1$, $v = 0.6$)

A Formal Definition

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Definition

The **Apollonius boundary** is the set of points $\mathcal{B} = \{X \in \bigcup_{i=1}^{n} \mathcal{A}_i \mid \mathcal{M}(E,X) \cap \left(\bigcup_{i=1}^{n} \mathcal{A}_i\right) = \{X\}\}$, where $\mathcal{M}(E,X)$ denotes the set of points on the line segment with endpoints E (position of the evader) and X.

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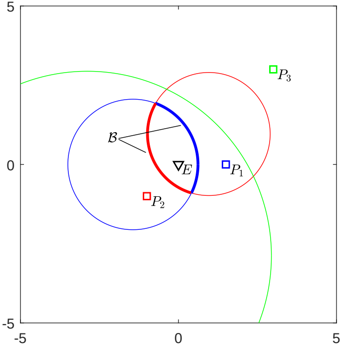
A Conjecture for the CB case

Conjecture

Pursuer P_i is active if $\mathcal{B} \cap \mathcal{A}_i \neq \emptyset$, and is redundant otherwise.



Lemma 1

Pursuer P_i is **the only active pursuer** if and only if

$$\mathcal{A}_i \cap \left(\bigcup_{j=1,\, j\neq i}^{n} \mathcal{A}_j \right) = \emptyset, \tag{1}$$

$$\mathcal{M}(E, T_i) \cap \left(\bigcup_{j=1,\, j\neq i}^{n} \mathcal{A}_j \right) = \emptyset, \tag{2}$$

T_i is the closest point to the evader on the Apollonius circle $\mathcal{A}_i$

Lemma 2

Assumption: $\mathcal{A}_i$ intersects one or more of the other Apollonius circles.

P_i is an active pursuer if and only if there exists at least one $X \in \mathcal{I}_i$ such that:

$$\mathcal{M}(E, X) \cap \left(\bigcup_{j=1}^{n} \mathcal{A}_j \right) = \{X\}, \tag{3}$$

$\mathcal{I}_i$ is the set of intersections points between $\mathcal{A}_i$ and the rest of the Apollonius circles.

Algorithm to Identify Pursuer Status

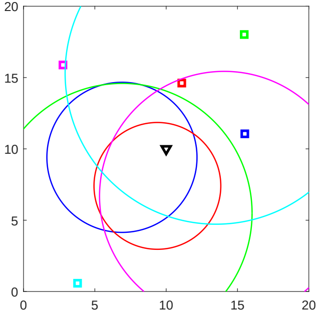
Algorithm 1 Obtain the status of pursuer P_i in the case of CB

Require: Positions of all the players $(p_1,\dots,p_n,p_E,i)$

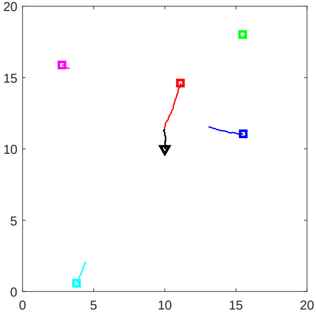
Ensure: Status of pursuer P_i

procedure OBTAIN_STATUS$(p_1,\dots,p_n,p_E,i)$
    flag1 $= 0$ *(To check if $\mathcal{A}_i$ intersects any other Apollonius circle)*
    status = redundant
    for $j = 1$ to n and $j \neq i$ **do**
        Obtain $\mathcal{I}_{ij}$ *(set of intersection points (X_ℓ) for $\mathcal{A}_i$ and $\mathcal{A}_j$)*
        if $\mathcal{I}_{ij} \neq \emptyset$ **then**
            flag1 $= 1$
            for $\ell = 1$ to card$(\mathcal{I}_{ij})$ **do**
                flag2 $= 0$. *(To check if $\mathcal{M}(p_E, X_\ell)$ intersects any other Apollonius circle)*
                for $k = 1$ to n and $k \neq i, j$ **do**
                    if $\mathcal{M}(p_E, X_\ell)$ intersects $\mathcal{A}_k$ **then**
                        flag2 $= 1$
                if flag2 $= 0$ **then**
                    status = active
                    break from outermost loop.
    if flag1 $= 0$ **then**
        status = active
        for $j = 1$ to n and $j \neq i$ **do**
            if $\mathcal{M}(p_E, T_i)$ intersects $\mathcal{A}_j$ **then**
                status = redundant
                break
    return status

Simulations



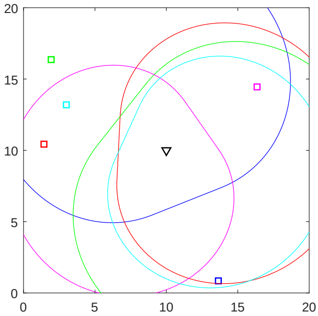
(a) Initial Apollonius circles



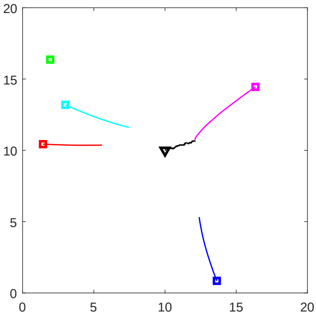
(b) Trajectories

Figure 8: CB case

Simulations



(a) Initial Apollonius curves (refined)



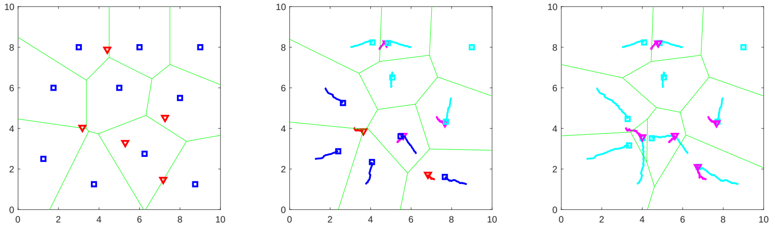
(b) Trajectories

Figure 9: PP case

An extension to multi-evaders case

1. Find the set of all active pursuers for each evader
2. Check if each active pursuer is assigned to a single evader
3. Break the tie by assigning the closest evader
4. Obtain the set of unassigned pursuers
5. Add the unassigned pursuers to the current assignment, and recheck active pursuers
6. Repeat steps (3)-(5) until (2) is satisfied

Simulations



(a) $t = 0$ (b) $t = 1.3$ (c) $t = 2.5$

Figure 10: CB case

Future Work

- Estimate when the assignment can change to avoid unnecessary calculations
- Account for turn-radius constraints on the players